GÖTTERZAHLEN AND SCALE STRUCTURE

By RICHARD DUMBRILL

The Neo-Babylonian text UET VII, 126[1] , published by Professor O.R. Gurney, shows that in l.4[2] a fourth-string is listed bilingually as Sumerian sa.4.tur, 'fourth, small-string', and Akkadian *a-ba-nu-[ú]*, 'Ea-created'. It is the only string in the text which is qualified by the name of a deity. The pairing of Ea with this fourth string, whilst the god is usually associated with the number 40[3] , attracted my attention in the light of a line in ASKT, p. 128, rev. 5f., where Ištar praises herself in the following terms: sa.a i.bí mèn sa.a a.ba mèn LI *mahru anaku* LI *arku anaku*. But there I had hoped that 'LI', in the Akkadian translation of the Sumerian phrase would have equated to Sum. sa = *pitnu* (or *pidnu*[4]), 'string', as had been thought until recently[5] , thus translating as "I am the first string, I am the last string[6]" . However, Dr George further brought to my attention that a new reading of the controversial 'LI', as Akkadian *še-ta*[7] 'net', compromised seriously the validity of my assumption. But this does not hinder the fact that being associated with the number 15 the goddess remains an entity comparable to Ea's quantity and that the two deities can be expressed as the ratio of 40/15. So despite the erroneous reading of 'LI' we still have evidence of two gods which not only associate with numbers but also with musical quantities as I shall now demonstrate.

Now the ratio of their respective numbers, 15 and 40, should reveal some musical coherence which in turn should be echoed with other 'Hauptgötter-numbers' such as 60[8] from Anu, (also 21[9]) ; 50 for Enlil, 30 for Sin, 20 for Šamaš, 10 for Bel Marduk, 14[10] for Šakan and Nergal and 6 for Adad. My assumption is that, in a musico-mythographical context, god numbers would have expressed either units of frequencies or units of string lengths, or both, as shown below, from Ištar to Anu:

15 30 40 50 60

1. If these figures express units of frequencies, then the highest note would be Anu's with 60 and the lowest, Ištar's with 15. Their ratio, 60/15, can be reduced to 4/1; the ratio of Anu (60) to Sin (30), 60/30 = 6/3 = 2/1, and that of Sin (30) to Ištar (15). 30/15, also equals 2/1. Since the doubling of the frequency number equates to the octave interval, then the distance between Ištar and Anu is two octaves. Sin stands right in the middle, an octave higher than Ištar and one lower than Anu.

2. If these figures express units of string lengths, then Anu is, with 60 units, the longest string, the bass note. Sin is one octave below Ištar and one above Anu. The ratios of string lengths are thus in reciprocal relation to the ratios of frequencies.

It seems appropriate at this point to introduce the musical cent or centième since it is the most tangible unit of tonometry. The conversion of ratios into musical cents consists in multiplying the log to base 10 of the quotient of the division between the denominator and numerator of the ratio by the constant 3986.314. This method produces a scale composed of 1200 units in which equal semitones measure 100 cents. Thus, 1/1 = 0 cents; 2/1= 1200 cents, the octave; 9/8 = 204 cents, the Pythagorean tone; 3/4 = 498, the just fourth; 2/3 = 702, the just fifth, etc.

From this we see that the gods' respective numbers are contained in the span of the top octave. Anu, Enlil, Ea and Sin provide with the tonal infrastructure for the Babylonian scale as shown below:

SIN EA ENLIL ANU

0 498 884 1200

Fundamental Fourth Sixth Octave

Anu/Enlil 60/50 = 6/5 = 316 = just minor third

Enlil/Ea 50/40 = 5/4 = 386 = just major third

Ea/Sin 40/30 = 4/3 = 498 = just fourth

Sin/Šamaš 30/20 = 3/2 = 702 = just fifth

Šamaš/Bel 20/10 = 2/1 = 1200 = octave.
With regards to the four gods contained in the upper octave, it will be interesting to note that their numbers produce the same intervals as those nomeclated in CBS 10996. This is the reason behind the presence of only two intervals, thirds and fifths in that text.

I would like to point out that these numbers, in a musicological context, could only have originated from empiricism applied exclusively to a fretted instrument such as the lute. The reason for this is simple enough as any unfretted paradigm such as a harp or a lyre, would not have allowed for any comparative metrology for the reason that frequency variations on such instruments result not only from the length, but also from variation in the tension and mass of their strings. With the lute, variations of frequencies are defined only by the accurate positioning of frets or of fret marks on the neck of the instrument, providing for the basic parameters for the elaboration of ratios.

When I first drafted this paper, there was a major obstacle to my thesis as the oldest iconographic representation of a lute dated from two seals of the Akkadian period, BM 89096 and BM 28806. There was a discrepancy between the period of the origins of the theory and the first apparition of a lute. This would have implacably disposed of my thesis, but it is only recently that Dr Dominique Collon acquired on behalf of the British Museum, a seal cylinder of the Uruk period on which there is an incontestable representation of a musician playing the lute. The seal is cataloged as BM WA 1996-10-2,1, and predates the aforementioned ones by some 800 years, reconciliating the periods for theory and practice, to

my greatest relief and satisfaction.

Later, the Greeks established their system around other numbers which I believe exempt of animism. Theirs were 12-9-8-6. The Babylonian system fits in as its series 6-5-4-3 precedes the Greek system in the series: 12-9-8-6-5-4-3. If the Babylonian figures are multiplied by 2 for the purpose of comparativity, then we have 12-10-8-6. It is the Babylonian value of 10 versus the Greek 9 which is essential to the distinction between both systems. Whilst the Babylonians attached more importance to minor and major thirds, 6/5 and 5/4, respectively, the Greeks looked on the fourth 12/4 and the tone 9/8.

Professor O.R. Gurney originally objected to my thesis on a purely Assyriological standpoint stating that the god-numbers were all part of a kabbalistic system that was invented in the Middle Assyrian period (c. 15th century BC) and has nothing to do with music. But in a post scriptum to one of his letters to me, Gurney states that Middle Assyrian scribes were merely adopting an older Babylonian system, which itself might have had earlier forgotten sources in the Uruk period since the principle of the lute, which is that of the principle of fretting would not have existed in the ignorance of the fundamental Götterzahlen.

Now the question as to which preceded the other remains: were the god numbers at the origins of the Uruk fret system of the lute or were the fret ratios at the origins of the god-numbers? I am willing to give preference to the second option as I am presently animated by subjectivist obnubilation.

Footnotes:

1 This text is best identified as a Late Babylonian manuscript of tablet XXXII of the series *Nabnitu*. See Finkel, I.L., **Materials for the Sumerian Lexikon** (MSL) XVI, 251; Wulstan, D., **The Tuning of the Babylonian Harp**, IRAQ XXX, (1968), 215-228; also **The Earliest Musical Notation**, MUSIC AND LETTERS 52, (1971), 365-382;

Duchesne-Guillemin, M., **A Hurrian Musical Score from Ugarit: The Discovery of Mesopotamian Music**, SANE, Vol. 2, Fasc. 2, (Malibu 1984) 5-24; also **Survivance Orientale dans la Désignation des Cordes de la Lyre en Grèce**, SYRIA 44, (1967), 233-246; also **A l'Aube de la Théorie Musicale: Concordance de trois tablettes Babyloniennes**, REVUE DE MUSICOLOGIE 52, (1966) 147-162; Crocker, R., and Kilmer, A.D., **The Fragmentary Text from Nippur**, IRAQ VLVI, (1984), 81-85; also **A Music Tablet from Sippar**, IRAQ XLVI, Part 2 (1984), 69-79; Vitale R., **La Musique Suméro-Accadienne**, UGARIT-FORSCHUNGEN, (1982), 241-263; West, M.L., **The Babylonian Musical Notation and the Hurrian Musical Texts**, MUSIC AND LETTERS 75/4 (1993), 161-179.

2 Dr A.R. George having read the manuscript of the present paper brought to my attention the fact that the 4th line of the Akkadian column of Professor O.R. Gurney's VIIth tome of the **Ur Excavation Texts** (1973), Plate LX, only shows *A* and not *A-ba-nu[ú]* (Kilmer), or A.[DÙ] (Finkel). I subsequently wrote to Gurney who replied to me that the tablet, now in Baghdad, read *a-ba-nu-[ú]* but that he inadvertently had left the '*-ba-nu-[ú]*' out. The error is corrected in IRAQ XLVI, 82, fn. 1.

3 Röllig, W., **REALLEXIKON DER ASSYRIOLOGIE** sub Götterzahlen, p. 499-500; Livingstone, A, **Mystical and Mythological Explanatory Works of Assyrian and Babylonian Scholars**, (Oxford, 1986), 30-49; Parpola, S., **The Assyrian Tree of Life: Tracing the Origins of Jewish Monotheism and Greek Philosophy**, JNES, 52 N.3, (1993), 182-4, fn. 86-89.

4 Kilmer, A.D., **The Strings of Musical Instruments: Their Names, Numbers and Significance**, AS 16, (1965), 262.

5 **SUMERIAN DICTIONARY** A/1 (Philadelphia, 1992), 69.

6 Kilmer, A.D., opus cit. The Strings . . ., 265.

7 Volk, K., **Die Balag-Komposition *úru-àm-ma-ir-ra-bi*** (Freiburger altorientalische Studien 18), (Stuttgart, 1989). Dr George, in a recent communication wrote to me that: 'According to Volk, the sign that Haupt read as LI , which never meant anything, should now be read '*še-ta*', 'net', i.e., nothing to do with stringed instruments. In fact, the Sumerian line can also be translated this way, because the Sumerian for 'net' and 'string' was the same (the sign SA depicts a mesh of strings) and that language does not distinguish between adverbs and adjectives. While we are not absolutely obliged to accept that a Babylonian's rendering of any given line of Sumerian is infallible, it should be noted that the two following lines also present the image of the goddess as a net, and that in those lines this image is unequivocal in both versions of the bilingual line.'

8 Parpola, *opus cit*. argues that the number for ANU was 1 and not 60 on the grounds that 60 does not make sense in his thesis and his subsequent 'tree demonstration' (p. 183). However, there is no dispute for the vertical wedge to be read as both 1 and/or 60, and if the other gods: ENLIL, EA and SIN, procede numerically, usually as 50, 40 and 30 respectively, then 60, for ANU makes complete sense. The sequence 1-50-40-30 is, one must admit, less convincing than 60-50-40-30. Furthermore, Parpola's observation that Ea's number was conceived as a sexagesimal fraction (40/60), would make ENLIL's as 50/60, (KINGUSILI *kingusili* = 5/6 '*parasrab*') 60 being ANU, and the same to apply for the whole pantheon. Furthermore, the principle of sexgasimalism present in the götterzahlen makes of ANU, the father of all gods, the ideal candidate for 60. He is the common denominator, common to all other gods because he is their father.The decimalisation of the sexagesimal fractions would have given: SIN = 30/60 = .5; ENLIL = 50/60 = .833, and ANU 60/60 = 1. The values of 60 and 1 for ANU are thus metrologically and mythologically identical.

9 See Neugebauer, O., JCS I (1947), 218; Labat, R., **Manuel d'Épigraphie Accadienne**, (Paris, 1948), 211: (d)21, le dieu Anu "in

colophons", (cf. *pa-lih* 21, 50 u 40, qui honore Anu, Enlil, Ea); Leichty, E., St. Opp., 152.

10 Livingstone, in opus cit. (fn. 3) has misread the number for žakan and Nergal as 11. This was corrected in King's copy (CT 25 50) as 14; Parpola, S., in opus cit. (fn.3).

THE URUK LUTE:

ELEMENTS OF METROLOGY

By RICHARD DUMBRILL

Earlier this year I examined a cylinder seal acquired by Dr Dominique Collon on behalf of the British Museum. The piece is now listed as BM WA 1996-10-2,1, and depicts, among others, the figure of a crouched female lutanist. The seal, which I shall not discuss here, has been identified by Dr Collon as an Uruk example and thus predates the previously oldest known iconographic representations by about 800 years. Little can be said about the instrument except that it would have measured about 80 centimetres long and that some protuberances at the top of its neck might be the representation of some device for the tuning of its strings. Otherwise, the angle of the neck in its playing position as well as the position of the musician's arms and hands is consistant with one of the aforementioned Akkadian seals, namely BM 89096. This shows that the instrument evolved very little for the best part of one millennium, for the probable reason that it already had completed its development, as early as the Uruk period.

The existence of the lute among the instrumentarium of the late fourth millennium is of paramount importance as it is consequential to the understanding and usage of ratios at that period. I am further willing to

hypothesize that the lute might have been at the origins of the proportional system. This is what I shall now demonstrate.

The lute differs from the two other types of stringed instuments, namely harps and lyres, in that each one of their strings produces more than one sound. This peculiarity qualifies the lute as a fretted instrument, not on the basis that it is provided with frets as we know them on the modern guitar, for instance, but in that each of the different notes generated from each of its strings is determined by accurate positions marked on the neck of the instrument. These are defined from the principle of ratios, and it is the principle of the stopping of the strings along the neck of the instrument that was at the origins of the understanding of such ratios.

DEMONSTRATION

Let us take a string that we stretch along the neck of a lute. Its length is measured from the bridge to the nut as this is the part of the string which vibrates freely to produce music. If a finger presses the string onto the neck of the lute at precisely half of its length, and if the half between the bridge and the finger is then plucked, the note heard will be one octave higher than it sounded when it was free. The ratio of the length of the free string to that of its half is therefore of 2:1. Babylonian musical theory defines two intervals, thirds and fifths. The position at which fingers should stop the strings to produce these intervals would be 6:5 for the minor third; 5:4 for the major third; and 4:3 for the fifth, and of course 6:3 = 2:1 for the octave. It goes without saying that we have there a series of ratios - 6:5:4:3 - which coincides with the god numbers of 60 for Anu, 50 for Enlil, 40 for Ea, and 30 for Sin. This is the basic infrastructure for the Babylonian scale. However, there is a problem since it is well attested that during the Old Babylonian period, the scales were incontestably descending, and not ascending as we have it on our Uruk lute. However, it can be argued that since ratios of string lengths behave in reciprocal relation to ratios of frequencies, that it all comes to the same. Furthermore, ratios can be inversed: minor thirds become major sixths; major thirds

become minor sixths; fourths become fifths and fifths become fourths etc. It is not improbable that despite of the fact that the approportionation of its strings was made on an ascending pattern, that the music played on the lute was composed in the characteristic style of a descending paradigm.

With regards to units of measurements in relation to the lute, it is possible to speculate that if the speaking length of a string equated to 2 KÙŠ, let us say 60 centimetres, since the average forearm is 30 cms, the octave fret would have been placed at 30 cms. If the other frets were then placed accordingly to this principle, the fret for the minor third, 6:5, would have been placed 10 centimetres away from the nut; the fret for the fifth, 4:3, would have been placed a further 10 centimetres down. If we take arbitrarily the free string as sounding an unspecified 'c', then the frets as placed above would produce c-eb-g-c, a chord of c minor. Now this numeric system is unable to provide with smaller intervals and it is possible that this is what decided theoreticians to increase the values of the gods to 60, 50, 40, 30 and so forth as it allowed for a more complex ratio system. This is well demonstrated with the ratio of Iëtar to Šakkan, 15:14, which is situated outside the pantheon of the higher gods (60-50-40-30). But if we make it fit in the pantheon, 15:14 becomes 60:56, and thus the fret for Šakkan would be placed 4 cms down from the nut and would produce 119,44 cents, a large semitone. In early Sumerian metrology, this would equate to 1 ŠU.SI. Now a ŠU.SI of 4 cms is large. It is commonly agreed that it would have measured 1.6 cms, at some point, and if we make it fit to our hypothetical system by means of approportionation, then the speaking length of the string would be 48 cms. If we allow for, let us say, 10 centimetres before the nut and 10 more after the bridge for the purpose of fixation, then the length of the whole instrument would be around 70 centimetres. This agrees with most of the iconographia.

Two other gods pair to produce the ideal complement to the ratio of Ištar to Šakkan (15:14): these are the second number for Anu, 21 and the number for Šamaš, 20. Their ratio 21:20 gives 84.47 cents, which added to the ratio of 15:14 = 119,44 cents, adds up exactly to 203.91 cents. This is

the so-called Pythagorean tone which equates to the Greek ratio of 9:8. I am willing to emphasize the argument that if up to now, the principal god numbers might have been coincidental with the fundaments of ancient musical theory, such sophisticated figures as the ones just debated cannot really confirm this coincidency. Thus, the figure of 119,44 cents would have been the Babylonian equivalent of the Greek apotome, and 84,47, the Babylonian equivalent of the Greek limma, but not any longer using the terms in their Greek definition because these Babylonian figures do not arise from the Greek methodology. The prevalence of the Babylonian apotome and limma, is futher made obvious when compared to their Greek equation which produce 114 cents for their apotome, corresponding to the ratio of 2048/2187, and 90 cents for their limma, corresponding to the ratio of 243/256.

It is clear, therefore that in spite of the fact that the Greek Pythagorean tone was a fundamental ratio in their harmonic series of 12:9:8:6, the structure of their semitones was not as simple as the Babylonian system.

THE MORPHOLOGY OF THE BABYLONIAN SCALE

By **RICHARD DUMBRILL**

In his last interpretation of cuneiform text UET VII 74, Professor O.R. Gurney has exposed the scales of the Old Babylonian musical system. The design of the present article is to analyse and define the origins and the morphology of the scale system in the early second millennium BC, in relation to the later Greek system, for the purpose of comparative musicology.

I

The Old Babylonian generative scale is as follows[1]:

[c]-b-a-g-f-e-d-c-b

This scale is called *išartum* and it starts from the upmost string of the *sammu*-instrument, string I, tuned as 'c^x'.

II

The second scale is produced by correcting the tritone placed on degrees V-II (f-b) of the first scale, *išartum*, by raising the fifth degree 'f', by one unqualified semitone[1]:

[c]-b-a-g-f$^\#$-e-d-c-b.

III

The principle of the correction of the tritones is then applied to each one of the resulting scales until it produces the following system:

1 c-b-a-g-f-e-d-c-b *išartum*

2 c-b-a-g-f$^\#$-e-d-c-b *qablitum*

3 c$^\#$-b-a-g-f$^\#$-e-d-c$^\#$-b *niš* GAB.RI

4 c$^\#$-b-a-g$^\#$-f$^\#$-e-d-c$^\#$-b *nid qablim*

5 c$^\#$-b-a-g$^\#$-f$^\#$-e-d$^\#$-c$^\#$-b *pitum*

6 c$^\#$-b-a$^\#$-g$^\#$-f$^\#$-e-d$^\#$-c$^\#$-b *embubum*

7 c#-b-a#-g#-f#-e#-d#-c#-b *kitmum*

8 c#-b#-a#-g#-f#-e#-d#-c#-b# išartum

Whilst these scales would have been played as they stand above, that is thetically, it is practical to mention that if transposed dynamically, they would read as:

1 c-b-a-g-f-e-d-c-b

2 f-e-d-c-b-a-g-f-e

3 b-a-g-f-e-c-d-b-a

4 e-d-c-b-a-g-f-e-d

5 a-g-f-e-d-c-b-a-g

6 d-c-b-a-g-f-e-d-c

7 g-f-e-d-c-b-a-g-f

8 c-b-a-g-f-e-d-c-b

It is possible, however that on an instrument fitted with 15 strings and tuned as :

c-b-a-g-f-e-d-c-b-a-g-f-e-d-c

. . . they would have offered the whole system, in the dynamic arrangement, but it is self evident that instruments which were restricted to thetical tuning would not have been compatible with those expanded to

the dynamical.

IV

The nine strings were numbered inwardly, as was indeed the case in ancient Greece. We can deduce that whilst the Greeks numbered seven distinct strings for the purpose of heptatonism, then if the Babylonians numbered nine, it is very likely that it was for the purpose of enneatonism. Furthermore, they needed nine distinctly named strings in order to generate a specific scale system, because, as it was produced by the technique of the correction of the tritones, they needed a span of nine notes to complement their cycle.

Thus the Old Babylonian system consisted in a diatonic descending enneatonic system which included seven distinct scales plus an eighth which, as the result of the method, consisted in the transposition of the first one, a semitone higher.

About the protogenesis of enneatonism, one must find an answer with pentatonism as had already been understood, although inconclusively, by Mme Duchesne-Guillemin. Pentatonism includes five distinct degrees of intervals no larger than a minor third and no smaller than a tone.

It is possible that the large zoomophic or the symmetric standing lyres of the Uruk period were played symmetrically by two players, facing each other, and that the fan shape of the string scaling would have revealed ascending scalings starting at each upright of the lyre with a hypothetical tuning as g-a-c-d-e-d-c-a-g, resulting in 9 distinct notes which would have obviously been numbered inwardly, as 1-2-3-4-5-4-3-2-1 since 1 = 1; 2 = 2; 3 = 3; 4 = 4, and that 5 was shared. The pentatonic system has no tritones. These would have appeared with diatonism. This led to the principle of their correction which in turn led to their specific scale system. Thus we can further state that enneatonism is the consequence of the adaptation of diatonism onto an older pentatonic system and that it is

the successive corrections of the tritone in each of the scales in which it appears which led to the generation of the Babylonian scalar system.

V

MORPHOLOGY

Professor Gurney has shown that the initial scale of the Old Babylonian system, *išartum*, equates to [c]-b-a-g-f-e-d-c-b. Most scholars have interpreted it as being the descending scale of c major. Since *išartum* is the initial scale of the cycle, then it should be the result of a natural tuning system. Now the only scale which can result from a fifths and fourths descending tuning method is the diatonic descending scale starting from b as b-a-g-f-e-d-c. They would have first tuned this b, then tuned fifth below, e, then fourth above, a, fifth below, d, fourth above, g, and fifth below, c, fourth above, f. Then they would have tuned another c at the upper octave, and another b, at the lower octave. The resulting paradigm consists of two junct pentachords: c-b-a-g-f, and f-e-d-c-b, the first is just, c-f, and the second is tritonic, f-b. The common note is 'f', string V. This further explains the reason behind symmetric notation. Now the instructions in UET VII 74 are to tune up this central fifth string/degree by the obvious quantity of an unqualified semitone. When this is done ('f' being 'f#'), we have in fact shifted the tritone at the upper pentachord (c-f, to c-f#). This system is fundamentally different from the later Greek system in which the scale is heptatonic and where the structure is the result of two junct tetrachords. The greek modal system differs from the Babylonian in that it is the positioning of the semitone in each tetrachord which dictates modality. In the Babylonian system, it is the position of the tritone (penta or tetrachordal) within the scale which determines modality. That both Babylonian and Greek methods lead to the same scales explains the evolution in the understanding of the harmonic cycle in both civilisations. Whilst the Babylonians saw it as a succession of pentachords, the Greeks saw it as a series of tetrachords.

VI

Epitome

The Old Babylonian scale system is the oldest recorded example. It is distinct from all others in that it is a descending diatonic enneatonic paradigm constituted of two junct pentachords. The quality of each scale is determined by the position of the tritone within the span of the enneachord. It is safe to assume that this system evolved into heptatonism during the first millennium BC. This is exemplified by CBS 10996 in which enneatonism only survives in the numbering of the strings. There, strings VIII = I and IX = II, but they do not add up to nine; they are considered within a heptatonic structure. It is not yet possible to determine if the renumbering of the strings to seven, in the same inwardly manner was the making of the Greek theoreticians, but it is now certain that Greek heptatonism found its origins in the Late Babylonian system.